Numerical Calculation of the Phase Space Density for the Strong-Strong Beam-Beam Interaction

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I. Mathematical model

- strong-strong beam-beam model (SSBB)
- beams are represented as densities Ψ, Ψ^* in four-dimensional phase-space $z = (x, p_x, y, p_y)$
- Two phase of evolution:
  a) on the orbit – "Rotate"
  b) at the interaction point (IP) – "Kick"
- Notation:
  n – revolution number (number of turns)
  Ψ_n, Ψ^*_n – densities before the kick
  Ψ_{n^+}, $\Psi^*_{n^+}$ – densities after the kick

- Evolution at IP is given by

$$\Psi_{n^+}(Kz) = \Psi_n(z)$$
$$K(x, p_x, y, p_y) = (x, p_x + k_x(x, y), y, p_x + k_y(x, y))$$
$$(k_x, k_y)^{\mathrm{T}} = \nabla\Phi^*,\ \Delta\Phi^* = \rho^*$$
$$\rho^*(x, y) = \int\int \Psi^*(x, p_x, y, p_y) dp_x dp_y$$

Remark. K^{-1} is trivial:

$$K^{-1}(x, p_x, y, p_y) = (x, p_x - k_x(x, y), y, p_x - k_y(x, y))$$

- Evolution on the orbit is given by

$$\Psi_{n+1}(Mz) = \Psi_{n^+}(z)$$
$$M = \begin{pmatrix} M_x & 0 \\ 0 & M_y \end{pmatrix}$$

II. Numerical Scheme

- Four-dimensional grid in phase-space.
- Densities are represented as four-dimensional arrays, which contain values of densities at grid-points.
- Coordinates of grid-points are tracked back. (Figure 2)
- Values of the densities between gridpoints are calculated using quadratic interpolation
- Poisson's equation is solved using conjugate-gradient algorithm.

III. Problems

- Large amount of data N^4. We need 100 millions gridpoints for only 100-point-per-dimension grid.
- So much data is hard to store and analyze. Solutions:
  a) implement visualization
  b) store and analyze means and covariance matrixes of distributions.

IV. CACHE' PROBLEM

- So much data cannot possibly fit into cache. This slows down computation dramatically.
- Solution of cache problem

$$\Psi_{n+1}(MKz) = \Psi_n \rightarrow \begin{cases} \Psi_{n+1/3}(z) = \Psi_n(K^{-1}z) \\ \Psi_{n+2/3}(z) = \Psi_{n+1/3}(\bar{M_x}^{-1}z) \\ \Psi_{n+1}(z) = \Psi_{n+2/3}(\bar{M}_y^{-1}z) \end{cases}$$

$$\bar{M}_x = \begin{pmatrix} M_x & 0 \\ 0 & I \end{pmatrix}, \quad \bar{M}_y = \begin{pmatrix} I & 0 \\ 0 & M_y \end{pmatrix}$$

- Two-dimensional layers easily fit cache.
- These calculations are easy to perform in parallel.

V. Code

- We developed code in C++ to calculate evolution of Beams that can be compiled such that computation is performed in parallel or in serial.
- The code can be compiled with or without graphics displaying. (This option now available for serial code only.)
- Precision doesn't have to be quadratic. It can be easily increased.

Beam energy (TeV)	7
Protons per bunch	$1.05 * 10^{11}$
β^*(m)	0.5
rms spot size at IP (μm)	16
Betatron tunes (ν_x, ν_y)	(0.31,0.32)